A SIMPLE ACOUSTIC IMMERSION INDEX FOR MUSIC PERFORMANCE SPACES

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A simple acoustic immersion index is proposed that compares the reverberant sound pressure level with the prompt sound pressure level, including both direct and once reflected contributions, for organ music in a hall defined only by its geometrical dimensions and reverberation time. Two versions of the index are considered. In the first version a very simple calculation is proposed that gives a constant index value S_1 throughout the hall. The second version of the index separates direct and once-reflected sound, with the result that the computed index S_2 varies from the front to the rear of the hall. Values of the index S_1 are given for several well known halls and are all close to 0 dB. The exact values appear to correlate well with subjective observations on the acoustical character of the halls.

INTRODUCTION

During the course of a 1998 ASA conference session and subsequent discussion [1] it appeared to me that, while organ builders and architects alike understand and appreciate the requirement for a balance between the clarity of an organ sound and the fullness contributed by reverberation, they do not have readily to hand any simple index by which this balance might be expressed. For existing halls, of course, they are able to present curves showing direct sound, prompt reflected sound, and reverberation, and there certainly exist means by which such curves can also be calculated at the detailed design stage, or measured in a model. Such studies are valuable, and indeed essential, but they require much detailed design work and subsequent analysis.

It seems that it would also be useful to have a simple index that gives a moderately reliable measure of the ratio of reverberant to prompt sound for a given hall and that can be worked out with very little labour or detailed design information. The 60 dB reverberation time T_{60} is indeed one such index that is commonly used, but it tells only part of the story. A brief search of standard works dealing with architectural acoustics [2-6] shows that the subject is well understood, as might have been expected, but no index parameter of simplicity comparable to the reverberation time appears to have been suggested. It is the purpose of this paper to propose such an index, which I will call an immersion index. In essence it measures the degree to which the listener feels immersed in the sound field, rather than perceiving it as coming from the general direction of the instrument.

Qualitatively, this immersion index is the inverse of various types of “clarity index” that have been proposed, which generally measure the ratio of directly propagated sound to reverberant sound at various positions in the hall [4]. This concept of clarity will be discussed again in a later section. For the present we simply note that the first-order immersion index discussed below has the great advantage of being a single number that is very simply calculable.

Because the sessions that provoked this response dealt with organ sound, and because this is a case that is relatively simple to analyse, it will taken as the basis for discussion. It is hoped, however, that a modified version of the index, or even the same index, might prove useful in preliminary assessment of performance spaces when used for other instruments or for choirs.

Organ sound is particularly suited to this discussion for several reasons. The first is that it is sustained, rather than percussive; the second is that the sound source, considered globally, is spatially distributed; and the third is that organ sound can be readily modified by adding or subtracting ranks of higher pitch. The perceptual attribute of immersion in organ music is also easier to define than in many other cases because of the nature of the music itself: sharp percussive transients are absent, and we deal instead with contrapuntal passages or with massed chords. The acoustical requirements placed upon the performance space are different in each case, but the proposed immersion index, along with the reverberation time, may perhaps serve to provide an adequate semi-quantitative initial descriptor.

A FIRST-ORDER INDEX

It is reasonable to take the reverberation time T_{60} of a performance space, supplemented by a knowledge of the enclosed volume V, as a zeroth-order measure of sound quality. Well-known curves [2] give ranges of reverberation time appropriate for particular types of music in halls of specified volume. What we now seek is a somewhat more refined index that measures, in some approximate sense, the relationship between early sound – that which is received within about 20 ms of the direct sound and is perceived as part of it – and the background of reverberant sound. We can think of this situation quite clearly in the case of contrapuntal organ music, because the input of sound energy to the space is approximately constant over a time much longer than the reverberation time, so that a steady reverberant state is achieved.

Consider the case of a simple rectangular “shoe-box” performance space of width W, height H and volume V, with the organ distributed over one end. In this first order approximation we neglect details of direct propagation and reflection and simply assume that all the sound power P of the organ is initially spread uniformly over the whole cross-section of the hall after at most a single reflection, so that the prompt intensity is

$$I_P = P/WH \tag{1}$$

It is assumed that reflection losses at surfaces near the front of the hall can be neglected.

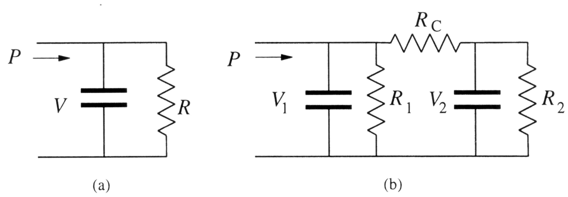

Fig. 1. (a) Energy-flow network analog for a simple reverberant space. (b) Energy-flow network analog for two coupled reverberant spaces.

The reverberant sound pressure can be calculated in a simple fashion, which we set out in some detail because it will be elaborated later. Fig. 1(a) shows an electric network analog for the acoustical problem, but in rather different terms from the usual analog. In this case the analog of electric current is taken to be the acoustic power P, and the analog of electric capacitance to be the enclosure volume V, so that the potential across the capacitance represents the acoustic energy density E. The electrical resistance R is proportional to the inverse of the acoustic absorption in the space, so that

$$R = \frac{K}{\sum \alpha_i A_i} \tag{2}$$

where A_i is the area and α_i the absorption coefficient of surface i and K is a constant that will be defined later. It is clear that the energy density for a constant acoustic power input P is given by $E_0 = PR$ and that, if this steady input is suddenly interrupted, the energy density will decay as $E_0 \exp(-t/\tau)$ where

$$\tau = RV = \frac{KV}{\sum \alpha_i A_i} \tag{3}$$

The resemblance between this equation and the normal Sabine reverberation- time relation is clear; the difference is that τ is the time for a decay of a factor e, which is equivalent to 4.343 dB, while T_{60} is the time for a decay of 60 dB, so that $T_{60} = 13.8\tau$. In the Sabine equation, τ is replaced by T_{60} and K by 0.163 $\mathrm{m^{-1}s}$ when metric units are used, so that comparison yields the value $K = 0.012\mathrm{m^{-1}s}$. The steady energy density in the enclosure for input power P is therefore

$$E_0 = \frac{0.012P}{\sum \alpha_i A_i} = \frac{0.074 T_{60} P}{V}\ . \tag{4}$$

The reverberant energy density given by (4) can be converted to a reverberant sound pressure p_{R} by the relation $E_0 = p_{\mathrm{R}}^2/\rho c$, where ρ is the density of air and c the speed of sound in air, while the corresponding relation for the prompt sound pressure is $I_{\mathrm{R}} = p_{\mathrm{P}}^2/\rho c$. Using (1) and the second form of (4), we can then define the first-order immersion index to be

$$S_1 \equiv 20\log_{10}(p_R / p_P) = 10\log_{10}(25WHT_{60} / V)\ \text{dB} \tag{5}$$

in which the numerical coefficient arises from substituting $c = 343\ \mathrm{ms^{-1}}$.

Extension of this index to halls that are not rectangular presents some problems that will be discussed later. For simple applications, it may be of value, however, to have a single number for the first-order index, even for halls that are not rectangular. This is easily derived by noting that the volume of the hall is $V = W\bar{H}L$, where L is the length of the hall and the over-bar indicates an average. With this convention

$$S_1 = 10\ \log_{10}(25T_{60}/L)\quad \text{dB} \tag{6}$$

This approximation applies to halls of any shape, provided they can be regarded as a single space rather than a set of coupled spaces.

EXAMPLES

It is useful now to calculate this first-order immersion index for a few well-known halls for which appropriate figures are readily available in the literature [4,5,7]. Table 1 shows relevant physical data for six well-known halls, together with numerical values for the index S_1. In each case the reverberation time quoted is an average over the 500–1000 Hz band with a full audience present.

There are several interesting features of the data in Table 1. In the first place, the immersion index is surprisingly close to 0 dB, indicating near equality between the reverberant sound level and the prompt sound level. Transients, however, will show up much more sharply and will generally not be masked by the existing sound. The index also refers to the frequency range 500–1000 Hz. At the higher frequencies up to say 6 kHz, characteristic of much organ sound, the reverberation time is reduced to about 60% of its 500 Hz magnitude, which decreases the value of S_1 by about 2 dB and gives the listener much more directional information. It is for this reason that octave ranks, mutations, and mixtures are so important in contrapuntal organ music. The rather small range of values of S_1 should be borne in mind when evaluating differences between acoustic environments.

The second notable feature is that the value of S_1 appears to correlate quite well with subjective assessment of the acoustics of the halls concerned. Royal Festival Hall, for example, is crisp and clear, while the Concertgebouw is warm and mellow [4]. The high value of S_1 for Cologne Cathedral similarly accords well with the listener's subjective feeling of immersion in the music. It should be noted, however, that the acoustics of such a large and reverberant cathedral, while excellent for general atmosphere, are perhaps not ideal for music except that specifically written for such buildings.

It is interesting, as an aside, to calculate the value of the index S_1 for a typical domestic bathroom, although the assumptions involved in its definition are not met in this case. The immersion index is about +4 dB, which is quite close to the value for a large cathedral. This perhaps explains the popularity of the bathroom environment with amateur tenors!

A SECOND-ORDER INDEX

The index proposed above suffers from one very clear defect, which is that it does not allow for the influence of direct sound but collects it into a more generalized ‘prompt sound’. It is possible to remedy this defect quite simply, but at the expense of additional complication in the calculation.

Referring to Fig. 2 for the case of a rectangular hall, the prompt sound received by a listener at point O can be divided into two parts, one of which is the sound propagated directly over a distance r and the other a more general early sound that has suffered a single reflection before reaching the listener. If I_{P1}, is the directly propagated intensity from a source of acoustic power P, which we take to be a small source such as the mouth of an organ pipe radiating isotropically into a half-space, then

$$I_{P1} = P/2\pi r^2 . \quad (7)$$

We must now determine the fraction of the source power that contributes to the more diffuse early sound that has suffered a single reflection before reaching the listener. To a reasonable approximation, only that sound that is reflected from the walls, ceiling or floor of the hall after travelling a distance less than $r/2$ along the hall meets this requirement – sound closer to the hall axis will either be experienced as direct sound or else be reflected to listeners nearer to the back of the hall. This is illustrated in the figure. If we define an equivalent circular hall cross-section of radius a so that $\pi a^2 = WH$, then the solid angle subtended at the source by the once-reflected sound that can reach the listener at the point O is about equal to $2\pi r(r^2 + 4a^2)^{-1/2}$, so that the intensity in the once-reflected prompt sound, assumed uniformly distributed over the hall cross-section, is

$$I_{P2} \approx \frac{(1-\alpha_1)rP}{WH(r^2+4a^2)^{1/2}} \approx \frac{(1-\alpha_1)P}{WH}\left(\frac{\pi r^2}{\pi r^2+4WH}\right)^{1/2} \quad (8)$$

where α_1 is the area-averaged absorption coefficient of the walls, ceiling and floor towards the front of the hall.

With this modification we can now define a second-order immersion index S_2 by the relation

$$S_2 \equiv 10\log_{10}\left(\frac{p_R^2}{p_{P1}^2+p_{P2}^2}\right)$$
$$= S_1 - 10\log_{10}\left[\frac{WH}{2\pi r^2} + (1-\alpha_1)\left(\frac{\pi r^2}{\pi r^2+4WH}\right)^{1/2}\right] \quad (9)$$

where S_1 is given by (5) or (6). Clearly the value of the index S_2 approaches that of index S_1 when $r > (WH)^{1/2}$, but for smaller values of r near the front of the hall the more refined S_2 is less than S_1 because of the increased contribution of direct sound. To aid in this comparison, Fig. 3 shows the quantity $S_2 - S_1$ as a function of the parameter $r/(WH)^{1/2}$. Specifically, $S_2 < S_1$ if $r < 0.52(WH)^{1/2}$ under the approximations we have adopted. For larger values of r, S_2 is always greater than S_1 because S_1 tends to overestimate the amount of once-reflected sound in the prompt sound.

It is clear that the index S_2 gives more information about the acoustics of the hall than does index S_1, though it requires rather more labour to calculate, display and evaluate. In particular, Fig. 3 displays the dominance of direct sound at the very front of the hall, and even suggests an optimum listening position near $r = (WH)^{1/2}$, which is generally about one-third of the distance from the front, at which the immersion index is high but, at the same time, the amount of direct sound is large, giving clarity. It is open to question, however, just how meaningful some of this extra information is, since it involves assumptions about wall and ceiling reflections that may well vary from one hall to another.

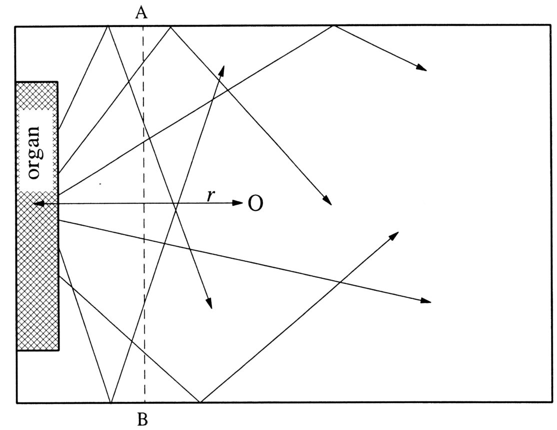

Fig. 2. Simple rectangular hall showing a listener at position 0, a distance r from the organ. Only rays that strike the walls or ceiling to the left of the plane AB contribute to the prompt reflected sound. Other rays may contribute to the direct sound, and all ultimately contribute to the reverberant sound.

The relation of index S_2 to various clarity indices that measure the ratio of directly propagated sound to reverberant sound in various parts of the hall is immediately apparent. The main distinction, apart from a change in sign, is that the prompt once-reflected sound is generally omitted or added into the reverberant sound. The intensity of the directly propagated sound, and thus the simple clarity index, therefore falls by 6 dB for each doubling of distance, so that the index does not achieve a saturation value. While this comment is not meant to denigrate the value of such a clarity index, the constant value of S_1 and the saturation behaviour S_2 confer desirable simplicity on these measures.

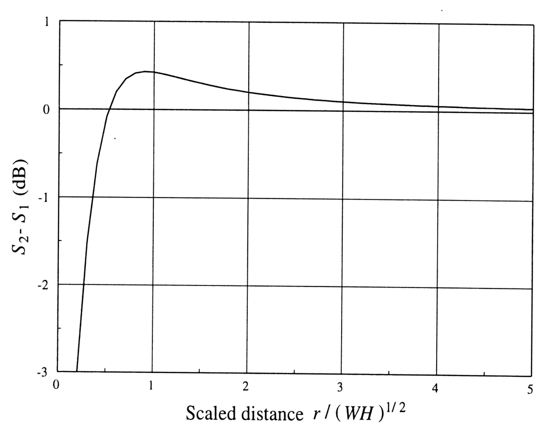

Fig. 3. Difference between the second-order immersion index S_2 and the first-order index S_1 as a function of distance r from the sound source, normalized in terms of the square root of the hall cross section WH.

NON-RECTANGULAR HALLS

So far our discussion has dealt specifically with rectangular halls, or at least halls of constant cross-section. While this class encompasses most concert halls, at least approximately, it is important to examine whether the index can be applied to halls of other shapes. Equation (6) has already suggested a way in which the first-order index might be calculated for a hall of general shape. It is tempting to go one step further and use equation (5), with the cross-sectional area WH taken as a function of distance from the front of the hall, to derive a spatially varying first-order index for a hall of arbitrary shape, but a trial calculation for a fan-shaped hall suggests that this overestimates S_1 near the front of the hall and underestimates it near the rear. It is probably necessary, when spatial variation of the index is being examined, to follow a course such as that adopted in the calculation of S_2.

In a hall with diverging side walls, the prompt wall reflections are directed much more towards the rear of the hall than they are in a rectangular hall. this has the effect of increasing the numerical coefficient 4 in the term $4WH$ in the denominator of equations (8) and (9) to a much larger value, depending upon the angle of divergence of the walls. While the walls contribute only part of the reflected sound, this modification will decrease the level of once-reflected sound near the front of the hall and increase its level near the rear, thus tending to equalize the index S_2, apart from the effect of direct sound in the immediate front of the hall. Few fan-shaped halls, however, have sidewalls that are simple planar reflectors, so that further detailed consideration, inappropriate for a simple index of the type proposed, is required.

The uncertainties involved in constructing appropriate modifications to the index S_2 in this way, however, bring into question its value as a design parameter in this case. For halls that are not well approximated by a simple rectangular shape, it may therefore be well to use only the simple uniform approximation (6) for S_1 and leave any more sophisticated index to the detailed design stage.

COUPLED VOLUMES

In many architectural designs, though perhaps not generally in concert halls, it is possible to consider the enclosure as consisting of two volumes more or less closely connected, rather than as a single volume. An example might be a rather long cathedral, with the nave linked to the chancel through a rather low or narrow tower crossing or, in the case of a concert hall, a reverberant enclosure purposely left behind or above the organ and coupled to the hall through a relatively small aperture. The network analog for such a situation is shown in Fig. 1(b). Each volume V_i can be considered as having an exponential decay constant $T^{(i)}_{60}$ that is derived by supposing the coupling aperture to be blocked by an ideal diffuse reflector. This information then defines the two resistive components R_i through the relation

$$R_i = T^{(i)}_{60} / (13.8 V_i). \tag{10}$$

The coupling resistance R_C is simply equal to the constant K of equation (2) divided by the area A_C of the coupling aperture. Thus

$$R_C = K / A_C = 0.012 / A_C \tag{11}$$

where metric units are assumed.

It is now straightforward to solve the network and calculate the energy densities E_1 and E_2 in both enclosures 1 and 2, assuming the sound source to be in enclosure 1. The results are

$$E_1 = \frac{(R_C + R_2) R_1 P}{R_C + 2R_1 + R_2}$$

$$E_2 = \frac{R_1 R_2 P}{R_C + 2R_1 + R_2} \tag{12}$$

and from these the reverberant sound pressures can be calculated using the relation $E = p_R^2/\rho c$ as before. The prompt sound pressure in enclosure 1 can be calculated just as for a simple enclosure, while the derivation for enclosure 2 follows the same path except that the power of the source is taken to be the prompt sound power $I_1 A_C$ entering through the coupling aperture, where I_1 is the prompt intensity at this aperture. Clearly there are additional problems if the aperture is in a side wall of enclosure 1, as would happen, for example, in the transepts of a cathedral, and this simple approach is then no longer adequate. Calculation of the immersion index for the two spaces now proceeds as before, and either S_1 or S_2 can be evaluated for each. The detail of the result for S_2 is too complicated to quote here, but for the first-order indices we find

$$S_1^{(1)} = \left(\frac{E_1 c W H}{P} \right)^{1/2}$$

$$S_1^{(2)} = \left(\frac{E_2 c W^2 H^2}{P A_C} \right)^{1/2} \tag{13}$$

where E_1 and E_2 are given by (12).

The analog network of Fig. 1(b) can also be used to calculate the form of the decay transient for an abruptly terminated sound input. It will consist, in general, of two superimposed exponential decays

$$A \exp(-t/\tau_1') + B \exp(-t/\tau_2') \tag{14}$$

where τ_1' and τ_2' are modified versions of τ_1 and τ_2, the extent of the modification depending upon the area A_C, of the coupling aperture. In enclosure 1, $A > B$, while in enclosure 2, $B > A$. The expressions for τ_1' and τ_2' can be calculated in a straightforward manner, but are algebraically complicated. Since this topic is of no immediate concern to us here, we shall not pursue the subject further.

DISCUSSION

It has been the purpose of this paper to propose a simply calculable index that has the potential to describe the sensation of auditory immersion of a performance space and thus to supplement other simple indices such as reverberation time and volume per seat. The first-order index has the advantage of being a single number, with approximate level 0 dB, that can be calculated immediately the volume, cross-section and reverberation time of the hall are known. The second-order index varies spatially throughout the hall and gives additional information that may be of use.

Comparison of the simple first-order index evaluated for several well-known concert halls containing organs, and also for a Gothic cathedral, suggests that it may indeed be useful as a preliminary guide for assessing the sound of an organ in a projected building, before going to the very much greater labour of making detailed acoustic calculations. The index also appears to give useful information in the case of much smaller spaces.

Because of its computational simplicity, the first-order index S_1 commends itself particularly for this purpose, when it is used together with other simple indices such as reverberation time. Only by practical trials can its usefulness be established.

REFERENCES

[1] *Architectural Acoustics: Acoustics of organ performance spaces.* I and II (abstracts only) *J. Acoust. Soc. Am.* **104**, 1823–1825 and 1840–1842 (1998)

[2] Knudsen V.O. and Harris C.M. *Acoustical Designing in Architecture*. Chs 8 and 9 (Acoustical Society of America, New York 1978)

[3] Kinsler L.E. and Frey A.R. *Fundamentals of Acoustics* 2nd Ed., Ch. 14 (John Wiley, New York 1962)

[4] Beranek L.L. *Music, Acoustics and Architecture.* (John Wiley, New York 1963)

[5] Meyer J. *Acoustics and the Performance of Music*. Ch. 6 (Verlag Das Musikinstrument, Frankfurt/ Main 1978)

[6] Beranek L.L. *Acoustics* 2nd Ed. pp.285–324, 425–429 (Acoustical Society of America, Woodbury NY 1986)

[7] Swaan W. *The Gothic Cathedral* pp. 118–127 (Crown / Park Lane, New York 1981)

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Table 1. First-order immersion index for some well-known halls

Hall	V (m^3)	W (m)	H (m)	T_{60} (s)	S_1 (dB)
Symphony Hall, Boston	18,700	23	19	1.8	+0.2
Grosser Musikvereinssaal, Vienna	15,000	20	18	2.05	+0.3
Herkulessaal, Munich	13,600	22	16	1.8	+0.7
Royal Festival Hall, London	22,000	33	16	1.47	–0.6
Concertgebouw, Amsterdam	18,700	29	18	2.0	+1.4
Cologne Cathedral	230,000	L =	120 m	13	+4.3